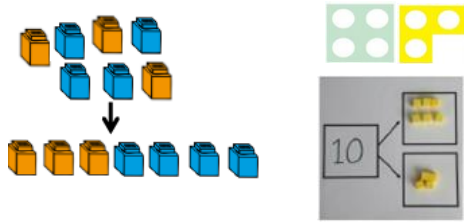


Addition

Counting and Combining sets of Objects

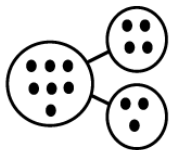
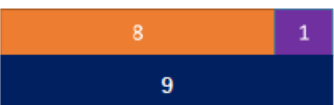
Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)

Concrete

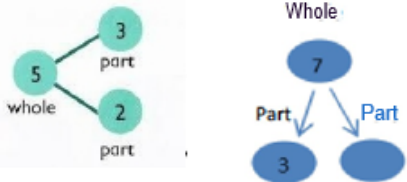


Pictorial

$8 + 5 = 13$



Abstract



$3 + 2 = 5$

$7 = 3 + \square$

Understanding of counting on

Starting with the larger number and counting on to find a total.

Concrete

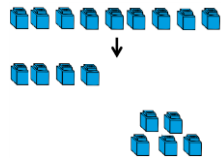


Subtraction

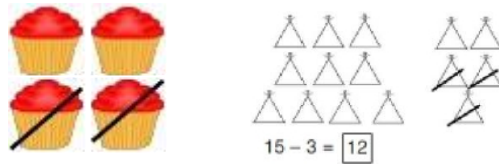
Taking away ones

Physically taking away and removing objects from a whole.

Concrete

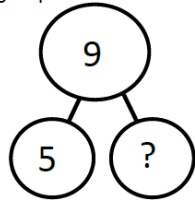


Pictorial



Abstract

$9 - 5 = 4$



Understanding of counting back

Starting with the larger number and counting on to find a total.

Concrete



Multiplication

Multiplication related to doubling

Concrete

Use a range of practical resources: Numicon, bead strings, multi-link etc...

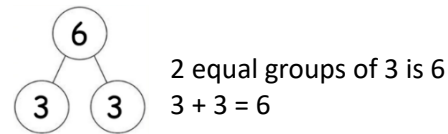


Pictorial

Draw pictures to show doubling



Abstract



Multiplication is related to groups of the same size (with support)

Concrete

Use a range of practical resources: Numicon, bead strings, multi-link and explain equal groupings

There are 3 equal groups with 4 in each group.



groups of 10

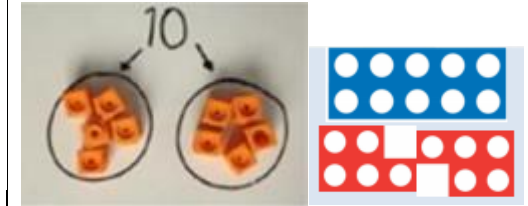
There are 4 equal groups of 5
There are 2 equal

Division

Division related to halving

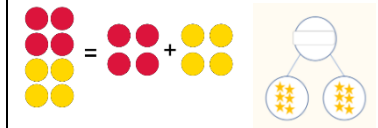
Use a range of practical resources: Numicon, bead strings, multi-link etc...

Concrete

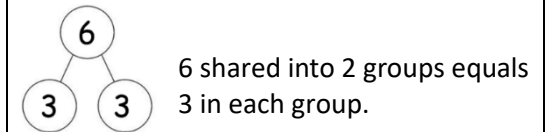


Pictorial

Draw pictures to show halving



Abstract

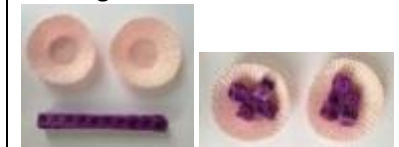


Group and share small quantities - understanding the difference between the two concepts.

Use a range of practical resources: Numicon, bead strings, multi-link etc...

Concrete

Sharing

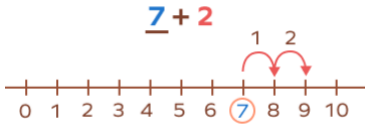


Grouping



Pictorial

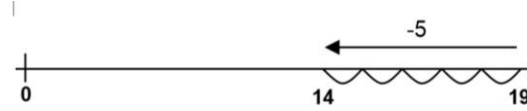
Pictorial



Abstract

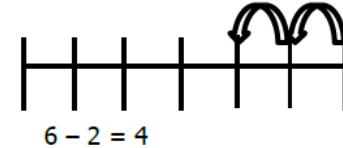
Counting on mentally

Pictorial



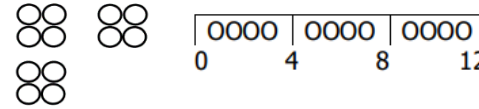
Abstract

Counting back mentally or using a blank number line for support.



3 hops of 4 is equal to 12

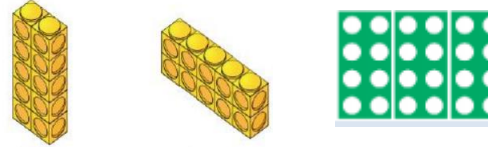
Pictorial



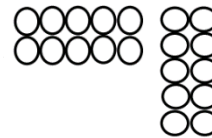
Use arrays to begin to understand that multiplication is commutative

Concrete

Use a range of resources to show arrays in different orientations.

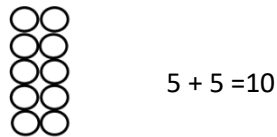
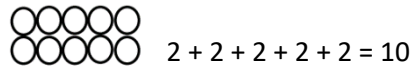


Pictorial



Abstract

Write the repeated addition calculations linked to the array.



Draw out objects to share or group and move to more structured pictorial representations such as number lines and bar models.

Sharing



Grouping



YEAR 2

Addition

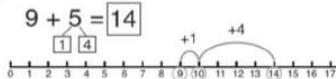
Partitioning and bridging through 10

Solve calculations with single digits that bridge 10 by regrouping.

Concrete



Pictorial



Abstract

$9 + \square = 14$

Use number bonds to make 10, then count on.

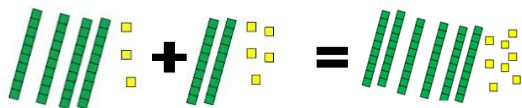
$9 + 1 = 10$

$10 + 4 = 14$

So: $9 + \square = 14$

Addition of 2-digit numbers without regrouping

Concrete



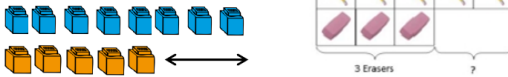
	T	O
+		

Subtraction

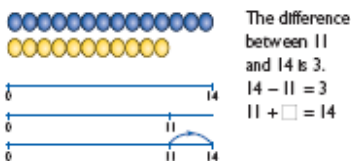
Understanding subtraction as finding the difference

Concrete

Calculate the difference between 8 and 5.



Pictorial

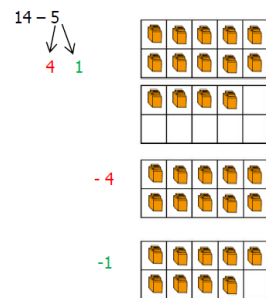


Abstract

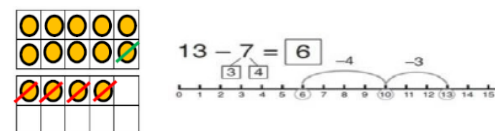
$8 - 5$, the difference is 3

Subtraction with bridging by making 10

Concrete



Pictorial

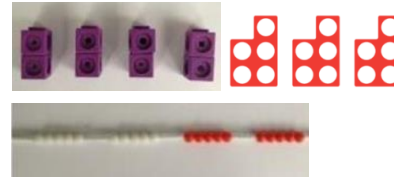


Multiplication

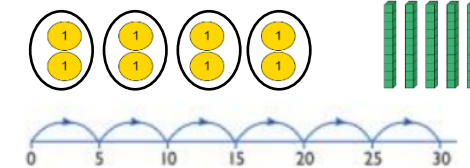
Counting in multiples

Concrete

Count in 2s, 5s and 10s using a range of practical resources.



Pictorial



Abstract

Write number sequences with multiples

2, 4, 6, 8, 10

5, 10, 15, 20, 25

Repeated addition and expressing multiplication as a number sentence using 'x'

Concrete

Represent given repeated addition and multiplication equations and solve using a range of practical resources



Division

Children should continue to use grouping and sharing for division using practical apparatus and pictorial representations learnt in Year 1.

Grouping using arrays

Use a range of practical resources: Numicon, bead strings, multi-link etc...

Concrete



Pictorial

Use of arrays as a pictorial representation for division.
 $15 \div 3 = 5$ There are 5 groups of 3.
 $15 \div 5 = 3$ There are 3 groups of 5



Abstract

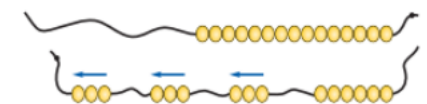
Know grouping- introducing children to the \div sign.

Grouping using a number line

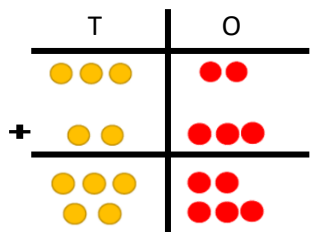
Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'

Concrete

Use bead strings or counting sticks.
 $15 \div 3 = 5$



Pictorial



Abstract

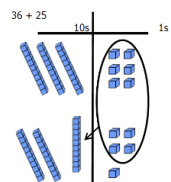
$$\begin{array}{r} 21 + 42 = \\ 20 + 40 \quad 1 + 2 \end{array}$$

$$\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$$

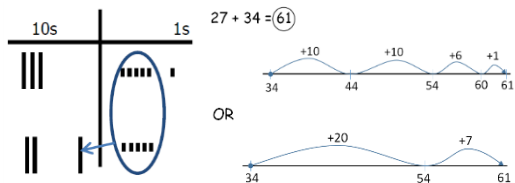
nb – when introducing methods linked to place value columns, always start with the ones for future learning.

Addition of 2-digit numbers with regrouping

Concrete



Pictorial



Abstract

$$\begin{array}{r} 36 + 25 \\ 30 + 20 = 50 \\ 5 + 5 = 10 \\ 50 + 10 + 1 = 61 \end{array}$$

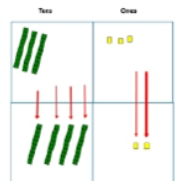
Abstract

$$14 - 5 = 9 \quad 14 - 4 = 10$$

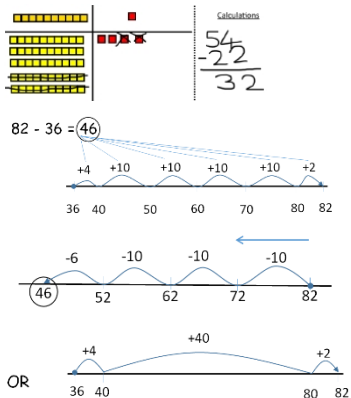
$$10 - 1 = 9$$

Subtraction of 2-digit numbers without exchanging

Concrete



Pictorial



Abstract

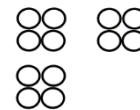
T	O
50	4
- 20	2
30	2

$30 + 2 = 32$

nb – when introducing methods linked to place value columns, always start with the ones for future learning.

Pictorial

Draw given repeated addition and multiplication equations to solve



Abstract

Write and solve equations demonstrating the link between repeated addition and using 'x' to show 'lots of' or 'groups of'.

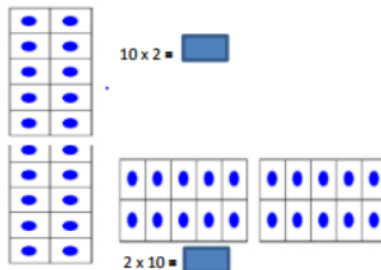
$$4 + 4 + 4 = 12$$

$$4 + 4 + 4 = 3 \times 4$$

$$3 \times 4 = 12$$

Develop understanding of commutativity by expressing arrays as multiplication number sentences

Concrete and pictorial



Abstract

Solve missing number problems

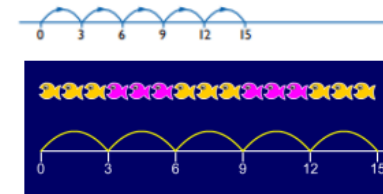
$$7 \times 2 = \square \quad \square = 2 \times 7$$

$$7 \times \square = 14 \quad 14 = \square \times 7$$

$$\square \times 2 = 14 \quad 14 = 2 \times \square$$

$$\square \times \bigcirc = 14 \quad 14 = \square \times \bigcirc$$

Pictorial



Abstract

Know grouping- introducing children to the ÷ sign.

Addition

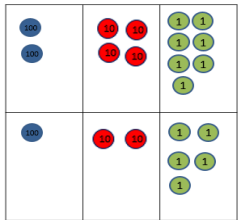
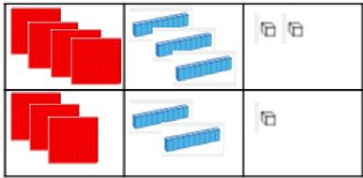
Mental methods

These should continue to develop, supported by a range of models and images, including the number line. Part-part-whole and bar models should continue to be used to help with calculating.

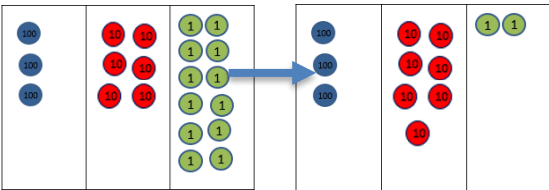
Addition of numbers with up to 3-digits using expanded column addition

Start with calculations without regrouping before introducing numbers that require regrouping in an expanded column method.

Concrete



Leading into the understanding of regrouping

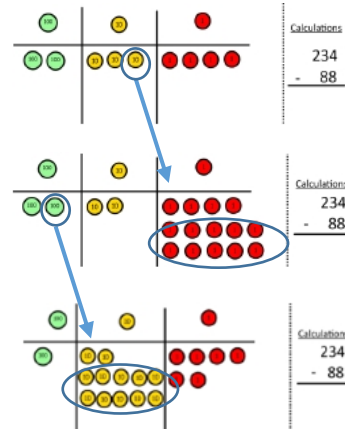


Subtraction

Subtraction of numbers with up to 3-digits using expanded column subtraction

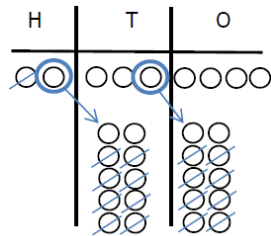
Start with calculations without regrouping with 3-digits to consolidate year 2 knowledge before introducing numbers that require regrouping in an expanded column method.

Concrete



Pictorial

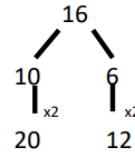
$234 - 188 =$



Multiplication

Mental methods

Doubling 2 digit numbers using partitioning



Multiplication of 2 digit by a 1 digit using partitioning

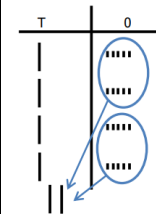
Concrete

Use resources to partition and rearrange $4 \times 15 =$



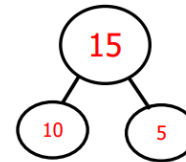
Pictorial

$4 \times 15 =$



Abstract

$4 \times 15 =$



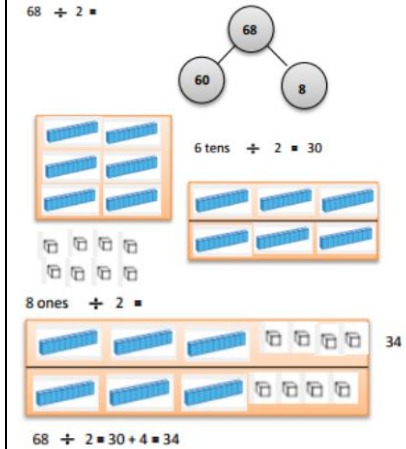
$10 \times 4 = 40$
 $5 \times 4 = 20$
 $40 + 20 = 60$

Division

Division using partitioning

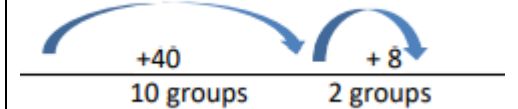
Becoming more efficient using a number line.

Concrete



Pictorial

$48 \div 4 = 12$



Abstract

$48 \div 4 =$

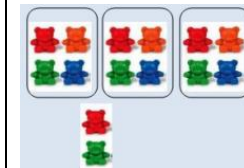
$40 \div 4 = 10$

$8 \div 4 = 2$

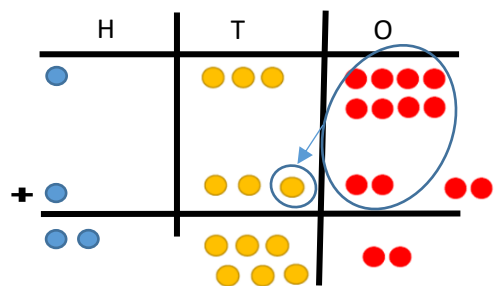
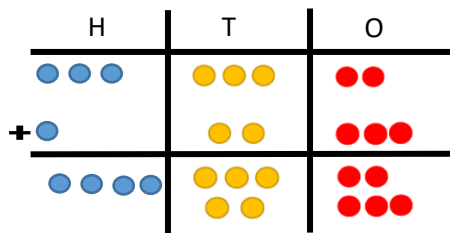
So, $48 \div 4 = 12$

Division with remainders

Concrete



Pictorial



Abstract

$$\begin{array}{r|l}
 400 & 90 & 4 \\
 + 300 & 60 & 8 \\
 \hline
 700 & 150 & 12
 \end{array}$$

$$\begin{array}{r|l}
 400 & 90 & 4 \\
 + 300 & 60 & 8 \\
 \hline
 800 & 60 & 2 \\
 100 & 10 &
 \end{array}$$

Abstract

Step 1

$$\begin{array}{r|l}
 700 & 20 & 3 \\
 - 300 & 40 & 6 \\
 \hline
 & &
 \end{array}$$

Step 2

$$\begin{array}{r|l}
 & 10 & 13 \\
 700 & 20 & 3 \\
 - 300 & 40 & 6 \\
 \hline
 & & 7
 \end{array}$$

Step 3

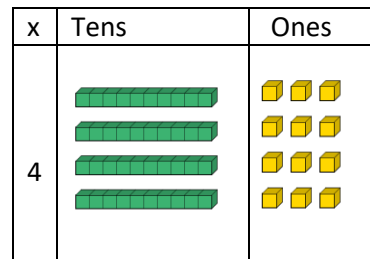
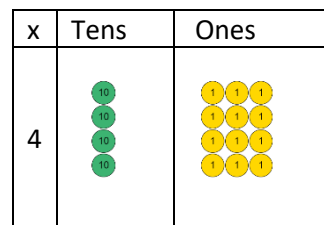
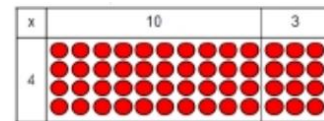
$$\begin{array}{r|l}
 600 & 110 & 13 \\
 700 & 20 & 3 \\
 - 300 & 40 & 6 \\
 \hline
 300 & 70 & 7
 \end{array}$$

Multiplication of 2 digit by a 1 digit using an informal written method – grid method

Concrete

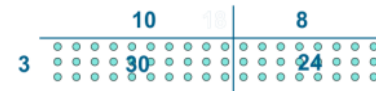
Use counters, place value counters and base 10 to represent calculations in a grid layout

$13 \times 4 =$

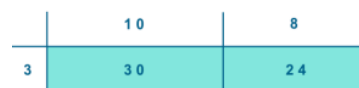


Pictorial

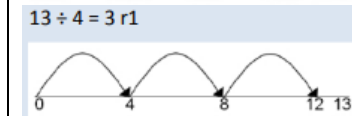
Represent the grid method by drawing versions of the concrete in books



Abstract



Pictorial



Abstract

$49 \div 4 =$

$40 \div 4 = 10$

$9 \div 4 = 2 \text{ r } 1$

So, $49 \div 4 = 12 \text{ r } 1$

Sharing – 49 shared between 4. How many left over? Grouping – How many 4s make 49? How many are left over?

YEAR 4

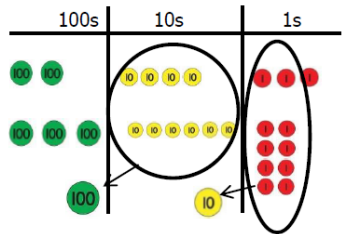
Addition

Mental methods

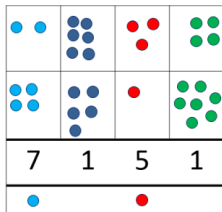
These should continue to develop, supported by a range of models and images, including the number line. Part-part-whole and bar models should continue to be used to help with calculating.

Addition of numbers with up to 4-digits using compact column addition

Concrete



Pictorial



Abstract

$$\begin{array}{r} 494 \\ + 368 \\ \hline 862 \\ 11 \end{array}$$

nb – apply previous steps to decimals in the context of money using coins and place value counters to support.

$$\begin{array}{r} \pounds 29.94 \\ + \pounds 4.37 \\ \hline \pounds 34.31 \\ 111 \end{array}$$

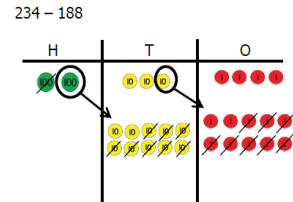
Subtraction

Mental methods

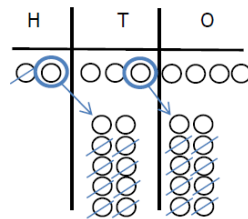
These should continue to develop, supported by a range of models and images, including the number line. Part-part-whole and bar models should continue to be used to help with calculating.

Subtraction of numbers with up to 4-digits using compact column subtraction

Concrete



Pictorial



Abstract

$$\begin{array}{r} \\ - \\ \hline \end{array}$$

nb – apply previous steps to decimals in the context of money using coins and place value counters to support

$$\begin{array}{r} \\ \pounds 27.126 \\ - \pounds 8.73 \\ \hline \pounds 18.53 \end{array}$$

Multiplication

Mental methods

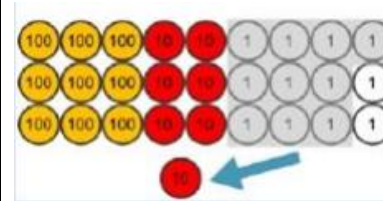
Counting in multiples of 6, 7, 9, 25 and 100, and steps of 1/10 and 1/100. Recall multiplication facts for multiplication tables up to 12 x 12. Use these facts to solve problems mentally.

Multiplication of 2 digit by a 1 digit using a formal written method (short multiplication)

Concrete

Use place value counters to solve calculations to model regrouping.

324 x 3 =



Pictorial

Represent using bar models, part-part-whole models and drawing counters.

$$\begin{array}{r} 972 \\ 324 \quad 324 \quad 324 \\ \hline 324 \times 3 \end{array}$$

Abstract

Expanded short multiplication (linking to grid method) moving towards the compact method.

$$\begin{array}{r} \\ \times \\ \hline \\ \\ \hline \end{array}$$

Division

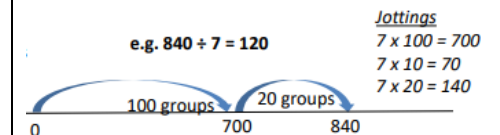
Sharing, grouping and using a number line using decomposition

Children will continue to explore division as sharing and grouping on a number line moving from partitioning to decomposition.

Concrete

Use a range of practical resources: place value counters, dienes blocks.

Pictorial



Abstract

840 ÷ 7 =

700 ÷ 7 = 100

140 ÷ 7 = 20

So, 840 ÷ 7 = 120

Formal Written Methods

Children should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Numbers should have **no** exchanging and regrouping initially until concept fully understood.
- Experiencing a logical progression in the numbers they use, for example: Dividend just over 10x the divisor, e.g. 84 ÷ 7

All of the above stages should include calculations with remainders as well as without.

YEAR 5

Addition	Subtraction	Multiplication	Division
<p><u>Mental methods</u> These should continue to develop, supported by a range of models and images learnt in previous years. Children should practise with increasingly large numbers to aid fluency</p> <p><u>Addition of numbers more than 4-digits using compact column addition</u> Using the same representations as Year 4, children will deepen their understanding of the compact column method and apply this knowledge to larger numbers.</p> <p><u>Addition of decimal numbers within the context of measure</u> When teaching measure, children will calculate with decimals using the compact column method where appropriate. Use representations from previous year groups to support understanding.</p>	<p><u>Mental methods</u> These should continue to develop, supported by a range of models and images learnt in previous years. Children should practise with increasingly large numbers to aid fluency</p> <p><u>Subtraction of numbers more than 4-digits using compact column subtraction</u> Using the same representations as Year 4, children will deepen their understanding of the compact column method and apply this knowledge to larger numbers.</p> <p><u>Subtraction of decimal numbers within the context of measure</u> When teaching measure, children will calculate with decimals using the compact column method where appropriate. Use representations from previous year groups to support understanding.</p>	<p><u>Mental methods</u> Mental methods X by 10, 100, 1000 using moving digits. Use practical resources and jottings to explore equivalent statements (e.g. $4 \times 35 = 2 \times 2 \times 35$). Solving practical problems where children need to scale up. Relate to known number facts. Identify factor pairs for numbers.</p> <p><u>Written methods (progressing to 4d x 2d)</u> Concrete Long multiplication using place value apparatus.</p> <p>Abstract Expanded long multiplication moving towards the compact method.</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="border: 1px solid #ccc; padding: 5px; margin: 5px;"> $\begin{array}{r} 32 \\ \times 24 \\ \hline 8 \quad (4 \times 2) \\ 120 \quad (4 \times 30) \\ 40 \quad (20 \times 2) \\ 600 \quad (20 \times 30) \\ \hline 768 \end{array}$ </div> <div style="border: 1px solid #ccc; padding: 5px; margin: 5px;"> $\begin{array}{r} 38 \\ \times 27 \\ \hline 266 \quad (38 \times 7) \\ 760 \quad (38 \times 20) \\ \hline 1026 \end{array}$ </div> </div> <div style="border: 1px solid #ccc; padding: 5px; margin: 5px; width: fit-content;"> $\begin{array}{r} 87 \quad \times \\ \hline 25 \\ 435 \\ \hline 1740 \\ \hline 2175 \end{array}$ </div>	<p><u>Formal Written Methods</u> Children should progress in their use of written division calculations:</p> <ul style="list-style-type: none"> Using tables facts with which they are fluent Experiencing a logical progression in the numbers they use, for example: <ol style="list-style-type: none"> Dividend just over 10x the divisor when the divisor is a teen number, e.g. $173 \div 15$ (learning sensible strategies for calculations such as $102 \div 17$) Dividend over 100x the divisor, e.g. $840 \div 7$ Dividend over 20x the divisor, e.g. $168 \div 7$ <p>Remainders should be interpreted according to the context. (i.e. rounded up or down to relate to the answer to the problem).</p> <p><u>Short division (progressing to 4d by 1d)</u> Concrete Short division to be modelled for understanding using place value counters.</p> <p>Pictorial Draw place value counters in books to support solving</p> <p>Abstract</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="border: 1px solid #ccc; padding: 5px; margin: 5px;"> $\begin{array}{r} 930 \div 3 \\ \hline 900 \quad 30 \end{array}$ </div> <div style="border: 1px solid #ccc; padding: 5px; margin: 5px;"> $\begin{array}{r} 30 \div 3 = 10 \\ 900 \div 3 = 300 \\ 930 \div 3 = 310 \end{array}$ </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 10px;"> <div style="border: 1px solid #ccc; padding: 5px; margin: 5px;"> $\begin{array}{r} 930 \div 3 = \\ \hline 900 \quad 30 \end{array}$ </div> <div style="border: 1px solid #ccc; padding: 5px; margin: 5px;"> $\begin{array}{r} 3 \overline{) 930} \\ \underline{- 900} \quad \rightarrow 300 \\ 30 \\ \underline{- 30} \quad \rightarrow 10 \\ 0 \end{array}$ </div> </div>

$$\begin{array}{r} 144 \\ 4 \overline{)576} \\ \underline{-400} \\ 176 \\ \underline{-160} \\ 16 \\ \underline{-16} \\ 0 \end{array}$$

	1	2	3
5	6	¹ 1	¹ 5

YEAR 6

Addition	Subtraction	Multiplication	Division																																																
<p><u>Mental and written methods</u> Mental methods should continue to develop, supported by a range of models and images with increased fluency. Children should be able to choose the most efficient method and explain why they chose it.</p> <p>Written methods should progress to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. Continue calculating with decimals, including those with different numbers of decimal places</p>	<p><u>Mental and written methods</u> Mental methods should continue to develop, supported by a range of models and images with increased fluency. Children should be able to choose the most efficient method and explain why they chose it.</p> <p>Written methods should progress to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. Continue calculating with decimals, including those with different numbers of decimal places</p>	<p><u>Mental and written methods</u> Mental methods X by 10, 100, 1000 using moving digits involving decimals. Identifying common factors and multiples of given numbers Solving practical problems where children need to scale up. Relate to known number facts.</p> <p><u>Written methods</u> Continue to refine and deepen understanding of written methods including fluency for using long multiplication</p>	<p><u>Formal Written Methods – long and short division</u> Continue to develop understanding of short division and introduce decimals.</p> <p><u>Long division (progressing to 4d by 1d)</u> Concrete Long division to be modelled for understanding using place value counters. Pictorial Draw place value counters in books to support solving Abstract</p> <table border="1" data-bbox="1682 667 2040 938"> <tr> <td></td> <td></td> <td></td> <td>2</td> <td>1</td> <td>2</td> </tr> <tr> <td>1</td> <td>2</td> <td>2</td> <td>5</td> <td>4</td> <td>4</td> </tr> <tr> <td></td> <td></td> <td>2</td> <td>4</td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td>1</td> <td>4</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td>1</td> <td>2</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>2</td> <td>4</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>2</td> <td>4</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td>0</td> </tr> </table>				2	1	2	1	2	2	5	4	4			2	4						1	4					1	2						2	4					2	4						0
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